CSX: An Extended Compression Format for SpMxV on Shared Memory Systems

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Compressed Sparse eXtended (CSX):
what: storage format for sparse matrices
why: optimize sparse matrix-vector multiplication (SpMxV) by (aggressively) compressing structural data
**background**
- sparse matrices
- the SpMxV kernel

**Compressed Sparse eXtended (CSX):**
what: storage format for sparse matrices
why: optimize sparse matrix-vector multiplication (SpMxV) by (aggressively) compressing structural data
Sparse matrices and sparse matrix vector multiplication
(application domain)

- Dominated by zeroes
- Applications: PDEs, graphs, linear programming
- Efficient representation: sparse storage formats
  (space and computation)
  - non-zero values (value data)
  - structural information (index data)
Sparse matrices and sparse matrix vector multiplication
(application domain)

- Dominated by zeroes
- Applications: PDEs, graphs, linear programming
- Efficient representation: sparse storage formats
  (space and computation)
  - non-zero values (value data)
  - structural information (index data)

- Sparse matrix vector multiplication (SpMxV)
  - \( y = A \cdot x \), \( A \) sparse
  - CG, GMRES, PageRank
  - considerable research attention*

*google scholar:
  - "sparse matrix vector multiplication" → 2280 results
  - "multicore" → 25100 results
CSR storage format
(Compressed Sparse Row)

\[
A = \begin{pmatrix}
5.4 & 1.1 & 0 & 0 & 0 & 0 & 0 \\
0 & 6.3 & 0 & 7.7 & 0 & 8.8 \\
0 & 0 & 1.1 & 0 & 0 & 0 \\
0 & 0 & 2.9 & 0 & 3.7 & 2.9 \\
9.0 & 0 & 0 & 1.1 & 4.5 & 0 \\
\end{pmatrix}
\]

- **row_ptr**: 
  \[0, 2, 5, 6, 9, 12\]

- **col_ind**: 
  \[0, 1, 1, 3, 5, 2, 2, 4, 5, 0, 3, 4\]

- **values**: 
  \[5.4, 1.1, 6.3, 7.7, 8.8, 1.1, 2.9, 3.7, 2.9, 9.0, 1.1, 4.5\]

- **nnz**: 
  \[\sum_{i=1}^{nrows} A_{ii} x_i + \sum_{i=1}^{nrows} A_{ii} y_i\]
CSR storage format
(Compressed Sparse Row)

\[
A = \begin{pmatrix}
5.4 & 1.1 & 0 & 0 & 0 & 0 & 0 \\
0 & 6.3 & 0 & 7.7 & 0 & 8.8 & 0 \\
0 & 0 & 1.1 & 0 & 0 & 0 & 0 \\
0 & 0 & 2.9 & 0 & 3.7 & 2.9 & 0 \\
9.0 & 0 & 0 & 1.1 & 4.5 & 0 & 0 \\
\end{pmatrix}
\]

\[
\text{row_ptr: } [0, 2, 5, 6, 9, 12, 15] \\
\text{col_ind: } [0, 1, 1, 3, 5, 2, 2, 4, 5, 0, 3, 4] \\
\text{values: } \{5.4, 1.1, 6.3, 7.7, 8.8, 1.1, 2.9, 3.7, 2.9, 9.0, 1.1, 4.5\}
\]
CSR storage format
(Compressed Sparse Row)

\[
A = \begin{pmatrix}
5.4 & 1.1 & 0 & 0 & 0 & 0 & 0 \\
0 & 6.3 & 0 & 7.7 & 0 & 8.8 & 0 \\
0 & 0 & 1.1 & 0 & 0 & 0 & 0 \\
0 & 0 & 2.9 & 0 & 3.7 & 2.9 & 0 \\
9.0 & 0 & 0 & 1.1 & 4.5 & 0 & 0
\end{pmatrix}
\]

\[
\begin{pmatrix}
x_0 \\
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5
\end{pmatrix}
= \begin{pmatrix}
y_0 = \sum A_{1i} \cdot x_i \\
y_1 = \sum A_{2i} \cdot x_i \\
y_2 = \sum A_{3i} \cdot x_i \\
y_3 = \sum A_{4i} \cdot x_i \\
y_4 = \sum A_{5i} \cdot x_i \\
y_5 = \sum A_{6i} \cdot x_i
\end{pmatrix}
\]

- \text{row_ptr:} 0, 2, 5, 6, 9, 12
- \text{col_ind:} 0, 1, 1, 3, 5, 2, 2, 4, 5, 0, 3, 4
- \text{values:} 5.4, 1.1, 6.3, 7.7, 8.8, 1.1, 2.9, 3.7, 2.9, 9.0, 1.1, 4.5
- \text{nrows+1: 6}
- \text{nnz: 12}
- \text{index data:}

\[
\text{index data:} \begin{pmatrix}
\end{pmatrix}
\]
CSR storage format
(Compressed Sparse Row)

\[
\begin{pmatrix}
5.4 & 1.1 & 0 & 0 & 0 & 0 & 0 \\
0 & 6.3 & 0 & 7.7 & 0 & 8.8 & 0 \\
0 & 0 & 1.1 & 0 & 0 & 0 & 0 \\
0 & 0 & 2.9 & 0 & 3.7 & 2.9 & 0 \\
9.0 & 0 & 0 & 1.1 & 4.5 & 0 & 0 \\
\end{pmatrix}
\begin{pmatrix}
x_0 \\
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
\end{pmatrix}
= \begin{pmatrix}
y_0 = \sum A_{1i} \cdot x_i \\
y_1 = \sum A_{2i} \cdot x_i \\
y_2 = \sum A_{3i} \cdot x_i \\
y_3 = \sum A_{4i} \cdot x_i \\
y_4 = \sum A_{5i} \cdot x_i \\
y_5 = \sum A_{6i} \cdot x_i \\
\end{pmatrix}
\]

Index data:
- row_ptr: 0 2 5 6 9 12
- col_ind: 0 1 1 3 5 2 2 4 5 0 3 4
- values: 5.4 1.1 6.3 7.7 8.8 1.1 2.9 3.7 2.9 9.0 1.1 4.5

nnz = number of non-zero elements
nrows + 1
CSR storage format

(Compressed Sparse Row)

\[ y_1 = x_1 \cdot 6.3 \]

\[
\begin{pmatrix}
5.4 & 1.1 & 0 & 0 & 0 & 0 & 0 \\
0 & 6.3 & 0 & 7.7 & 0 & 8.8 & 0 \\
0 & 0 & 1.1 & 0 & 0 & 0 & 0 \\
0 & 0 & 2.9 & 0 & 3.7 & 2.9 & 0 \\
9.0 & 0 & 0 & 1.1 & 4.5 & 0 & 0 \\
\end{pmatrix}
\begin{pmatrix}
x_0 \\
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
\end{pmatrix}
\]

\[
= \begin{pmatrix}
y_0 = \sum A_{1i} \cdot x_i \\
y_1 = \sum A_{2i} \cdot x_i \\
y_2 = \sum A_{3i} \cdot x_i \\
y_3 = \sum A_{4i} \cdot x_i \\
y_4 = \sum A_{5i} \cdot x_i \\
y_5 = \sum A_{6i} \cdot x_i \\
\end{pmatrix}
\]
CSR storage format
(Compressed Sparse Row)

\[ y_1 = x_1 \cdot 6.3 + x_3 \cdot 7.7 \]

\[
\begin{pmatrix}
5.4 & 1.1 & 0 & 0 & 0 & 0 & 0 \\
0 & 6.3 & 0 & 7.7 & 0 & 8.8 & 0 \\
0 & 0 & 1.1 & 0 & 0 & 0 & 0 \\
0 & 0 & 2.9 & 0 & 3.7 & 2.9 & 0 \\
9.0 & 0 & 0 & 1.1 & 4.5 & 0 & 0 \\
\end{pmatrix}
\begin{pmatrix}
x_0 \\
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
\end{pmatrix}
=
\begin{pmatrix}
y_0 = \sum A_{1i} \cdot x_i \\
y_1 = \sum A_{2i} \cdot x_i \\
y_2 = \sum A_{3i} \cdot x_i \\
y_3 = \sum A_{4i} \cdot x_i \\
y_4 = \sum A_{5i} \cdot x_i \\
y_5 = \sum A_{6i} \cdot x_i \\
\end{pmatrix}
\]
CSR storage format
(Compressed Sparse Row)

\[
\begin{align*}
y_1 &= x_1 \cdot 6.3 + x_3 \cdot 7.7 + x_5 \cdot 8.8 \\
A &= \begin{pmatrix}
5.4 & 1.1 & 0 & 0 & 0 & 0 & 0 \\
0 & 6.3 & 0 & 7.7 & 0 & 8.8 \\
0 & 0 & 1.1 & 0 & 0 & 0 \\
0 & 0 & 2.9 & 0 & 3.7 & 2.9 \\
9.0 & 0 & 0 & 1.1 & 4.5 & 0
\end{pmatrix} \\
\begin{pmatrix}
x_0 \\
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5
\end{pmatrix} \times \\
\begin{pmatrix}
y_0 \\
y_1 \\
y_2 \\
y_3 \\
y_4 \\
y_5
\end{pmatrix} &= \begin{pmatrix}
y_0 = \sum A_{1i} \cdot x_i \\
y_1 = \sum A_{2i} \cdot x_i \\
y_2 = \sum A_{3i} \cdot x_i \\
y_3 = \sum A_{4i} \cdot x_i \\
y_4 = \sum A_{5i} \cdot x_i \\
y_5 = \sum A_{6i} \cdot x_i
\end{pmatrix}
\end{align*}
\]

row_ptr:
0 2 5 6 9 12

col_ind:
0 1 1 3 5 2 2 4 5 0 3 4

values:
5.4 1.1 6.3 7.7 8.8 1.1 2.9 3.7 2.9 9.0 1.1 4.5

nnz

nrows+1

index data
parallel SpMxV for shared memory

- data partitioning
  - per rows

- load balancing
  - based on number of non-zeros

\[
\begin{align*}
A & \quad \times \quad x \\
\text{thread 0} & \quad \ast \quad \text{both threads} \\
\text{thread 1} & \quad = \quad \text{thread 0} \\
\text{thread 1} & \quad \ast \quad \text{both threads} \\
\end{align*}
\]
Traditional SpMxV optimization methods

- traditional goal: optimizing computation

- specialized sparse storage formats
  (exploitation of “regularities”)

- examples (regularity ↔ format):
  - 2D blocks of constant size ↔ BCSR [Im and Yelick ’01]
  - 1D blocks of variable size ↔ [Pinar and Heath ’99]
  - Diagonals ↔ DIAG
Traditional SpMxV optimization: BCSR

[Im and Yelick ’01]

- CSR extension: \( r \times c \) blocks instead of elements \( \Rightarrow \) per-block index information
- optimize computation (register blocking) \( \Rightarrow \) specialized SpMxV versions for \( r \times c \)

\[
A = \begin{pmatrix}
4.6  & 9.3 & 0 & 0 & 0 & 0 & 2.4 & 5.6 \\
8.6  & 8.2 & 0 & 0 & 0 & 0 & 5.3 & 1.6 \\
0 & 0 & 0 & 0 & 1.9 & 7.9 & 0 & 0 \\
0 & 0 & 0 & 0 & 7.1 & 0 & 0 & 0 \\
0 & 0 & 8.6 & 1.7 & 2.4 & 7.6 & 0 & 0 \\
0 & 0 & 3.9 & 2.2 & 3.0 & 3.3 & 0 & 0 \\
0 & 0 & 0 & 0 & 1.8 & 0 & 7.9 & 1.2 \\
0 & 0 & 0 & 0 & 0 & 7.8 & 1.0 & 5.3
\end{pmatrix}
\]

brow_ptr: \(0\ 2\ 3\ 5\ 7\)

bcol_ind: (0 \(\rightarrow\) 6 \(\rightarrow\) 4 \(\rightarrow\) 2 \(\rightarrow\) 4 \(\rightarrow\) 4 \(\rightarrow\) 6)

blocks:
\[
\begin{array}{cccccccc}
4.6 & 9.3 & 2.4 & 5.6 & 1.9 & 7.9 & 8.6 & 1.7 \\
8.6 & 8.2 & 5.3 & 1.6 & 7.1 & 0 & 3.9 & 2.2 \\
\end{array}
\]

bval: (4.6 \ 9.3 \ 8.6 \ 8.2 \ 2.4 \ 5.6 \ 5.3 \ 1.6 \ 1.9 \ 7.9 \ 7.1 \ 0.0 \ldots)
Traditional SpMxV optimization: BCSR

[Im and Yelick ‘01]

- CSR extension: $r \times c$ blocks instead of elements $\Rightarrow$ per-block index information
- Optimize computation (register blocking) $\Rightarrow$ specialized SpMxV versions for $r \times c$
- Padding may be required

$$A = \begin{pmatrix}
4.6 & 9.3 & 0 & 0 & 0 & 0 & 2.4 & 5.6 \\
8.6 & 8.2 & 0 & 0 & 0 & 0 & 5.3 & 1.6 \\
0 & 0 & 0 & 0 & 1.9 & 7.9 & 0 & 0 \\
0 & 0 & 0 & 0 & 7.1 & 0 & 0 & 0 \\
0 & 0 & 8.6 & 1.7 & 2.4 & 7.6 & 0 & 0 \\
0 & 0 & 3.9 & 2.2 & 3.0 & 3.3 & 0 & 0 \\
0 & 0 & 0 & 0 & 1.8 & 0 & 7.9 & 1.2 \\
0 & 0 & 0 & 0 & 0 & 7.8 & 1.0 & 5.3
\end{pmatrix}$$

brow_ptr: 0 2 3 5 7

bcol_ind: (0 6 4 2 4 4 6)

blocks: 4.6 9.3 2.4 5.6 1.9 7.9 8.6 1.7 2.4 7.6 1.8 0 7.9 1.2

bval: (4.6 9.3 8.6 8.2 2.4 5.6 5.3 1.6 1.9 7.9 7.1 0.0 ... )
SpMxV performance (CSR)

- related work \(\rightarrow\) several performance issues
- performance evaluation in 100 matrices [Goumas et. al. ’09]
- memory bandwidth is the bottleneck

\[^1\text{for matrices larger than cache}\]
SpMxV performance (CSR)

- related work → several performance issues
- performance evaluation in 100 matrices [Goumas et. al. ’09]
- memory bandwidth is the bottleneck \(^1\)

\(^1\)for matrices larger than cache
SpMxV performance (CSR)

- related work → several performance issues
- performance evaluation in 100 matrices [Goumas et. al. ’09]
- memory bandwidth is the bottleneck

> compression for improving SpMxV performance (reduce working set)

^for matrices larger than cache
CSX: approach

regularities and sparse storage formats

- BCSR, [Pinar and Heath ‘99], DIAG
- multiple regularities ↔ *composite formats* [Agarwal et. al ‘92]
  multiple sub-matrices — each in different format
  \[ A \cdot x = (A_0 + A_1) \cdot x = A_0 \cdot x + A_1 \cdot x \]
CSX: approach

regularities and sparse storage formats

- BCSR, [Pinar and Heath ’99], DIAG
- multiple regularities ↔ *composite formats* [Agarwal et. al ’92]
  - multiple sub-matrices — each in different format
  \[
  A \cdot x = (A_0 + A_1) \cdot x = A_0 \cdot x + A_1 \cdot x
  \]

(our) requirements

- support multiple regularities on the same matrix
- extendability – arbitrary regularities
- adaptability
CSX: approach

regularities and sparse storage formats

- BCSR, [Pinar and Heath ’99], DIAG
- multiple regularities ↔ composite formats [Agarwal et. al ’92]
  multiple sub-matrices — each in different format
  \[ A \cdot x = (A_0 + A_1) \cdot x = A_0 \cdot x + A_1 \cdot x \]

(our) requirements

- support multiple regularities on the same matrix
- extendability – arbitrary regularities
- adaptability

approach — CSX (Compressed Sparse eXtended) format

- units: matrix areas that adhere to a regularity
- unified detection of regularities
- code generation of specialized SpMxV routines
CSX outline

- CSX substructures (regularities)
- CSX detection of substructures
  - and how to make it faster
- Experimental evaluation
CSX substructures
(regularities supported by CSX)

- **Horizontal**

  
  \[
  \begin{array}{cccc}
  x & x & x & x & x \\
  \end{array}
  \]

  (e.g: col. indices: 1,2,3,4,5)

  sequential elements

  \[
  (y, x + i) \rightarrow (y, x) \ (y, x + 1) \ (y, x + 2) \ldots
  \]
CSX substructures
(regularities supported by CSX)

- **Horizontal** (delta run-length-encoding — drle)
  
  ![Horizontal pattern example](example)

  (e.g: col. indices: 2,4,6,8,10)

  sequential elements with a constant difference $\delta$

  $$(y, x + i \cdot \delta) \rightarrow (y, x) \ (y, x + \delta) \ (y, x + 2 \cdot \delta) \ldots$$
CSX substructures
(regularities supported by CSX)

- **Horizontal (delta run-length-encoding — drle)**

  Sequential elements with a constant difference $\delta$
  
  $$(y, x + i \cdot \delta) \rightarrow (y, x) \quad (y, x + \delta) \quad (y, x + 2 \cdot \delta) \ldots$$

- **Other 1D directions (Vertical, Diagonal, Anti-Diagonal)**
CSX substructures
(regularities supported by CSX)

- **Horizontal (delta run-length-encoding — drle)**
  
  Sequential elements with a constant difference $\delta$
  
  $$(y, x + i \cdot \delta) \rightarrow (y, x) \ (y, x + \delta) \ (y, x + 2 \cdot \delta) \ldots$$

- **Other 1D directions (Vertical, Diagonal, Anti-Diagonal)**

- **2D blocks**

  $$(x + i) \times (y + j) \text{ (double nested loop)}$$
CSX substructures on the matrix set

![Matrix substructures chart](image-url)
CSX substructures on the matrix set

ANTI-DIAG $\delta = 1$

DIAG $\delta = 11$
CSX substructure detection: horizontal

(Delta Run-Length Encoding – DRLE)

\[
\begin{pmatrix}
(1, 3) \\
(2, 1) (2, 2) (2, 3) (2, 4) \\
(3, 1) \\
(4, 3)
\end{pmatrix}
\]

(1, 3) (2, 1) (2, 2) (2, 3) (2, 4) (3, 1) (4, 3)
CSX substructure detection: horizontal

(Delta Run-Length Encoding – DRLE)

\[
\begin{pmatrix}
(1, 3) \\
(2, 1) & (2, 2) & (2, 3) & (2, 4) \\
(3, 1) \\
(4, 3)
\end{pmatrix}
\]

column indices: 1 2 3 4

deltas (\(\delta\)): 1 1 1 1

run-length-encoding: (\(\delta=1\), len=4)

- same order with storage \(\rightarrow\) detection is simple
CSX substructure detection: horizontal
(Delta Run-Length Encoding – DRLE)

\[
\begin{pmatrix}
(1,3) \\
(2,1) (2,2) (2,3) (2,4) \\
(3,1) \\
(4,3)
\end{pmatrix}
\]

- detection
  - column indices: 1 2 3 4
  - deltas (\(\delta\)): 1 1 1 1
  - run-length-encoding: (\(\delta=1\), len=4)

unit
- start: (2,1)
- order: HORIZ
- \(\delta\): 1
- length: 4

- same order with storage \(\rightarrow\) detection is simple
CSX substructure detection: generalization
CSX substructure detection: generalization

(Transformations)

\[
\begin{pmatrix}
(1, 1) & (1, 3) \\
(2, 2) & (3, 3) \\
(3, 3) & (4, 4)
\end{pmatrix}
\]

\[
\begin{pmatrix}
(4, 1) \\
(2, 1) \\
(4, 2) \\
(4, 3) \\
(4, 4)
\end{pmatrix}
\]

\[
i' = \text{nrows} + j - i
\]

\[
j' = \min(i, j)
\]

lexicographic sort

\[
(2, 1) (4, 1) (4, 2) (4, 3) (4, 4)
\]
CSX substructure detection: generalization

(Transformations)

\[
\begin{pmatrix}
(1, 1) & (1, 3) \\
(2, 2) & (3, 3) \\
(4, 4) & (4, 4)
\end{pmatrix}
\]

\[
i' = \text{nrows} + j - i \\
j' = \text{min}(i, j)
\]

\[
\begin{pmatrix}
(4, 1) & (2, 1) \\
(4, 2) & (4, 3) \\
(4, 4) & (4, 4)
\end{pmatrix}
\]

(1, 1) (1, 3) (2, 2) (3, 3) (4, 4)

(4, 1) (2, 1) (4, 2) (4, 3) (4, 4)

- add a regularity → provide transformation
CSX preprocessing phases

1. Detection: find and select substructures
2. Encoding:
   - index information stored in a byte-array
   - each unit: size (1 byte) type+markers (1 byte) payload
3. Code Generation: matrix-specific SpMxV routines generated programmatically using LLVM
   (code iterates substructures and perform the operation)
CSX preprocessing phases

1. Detection: find and select substructures
2. Encoding: 
   - index information stored in a byte-array 
   - each unit: size (1 byte) type+markers (1 byte) payload
3. Code Generation: matrix-specific SpMxV routines generated programmatically using LLVM
   (code iterates substructures and perform the operation)

→ what about preprocessing (compression) cost?
   ▶ depends on the application
   ▶ frequently, the matrix is used across numerous SpMxV runs
     • sufficient repetitions → overhead will be amortized
   ▶ methods to reduce preprocessing cost (in the detection phase)
     • tradeoff: performance vs preprocessing cost
reducing preprocessing cost
(and a more in-depth look at substructure detection)

**in:** `elems` (matrix elements)
**in:** `xforms` (set of transformations)

**while** `True` **do**
- `xf_{best} ← select_best(xforms,elems)`
- **if** `xf_{best} == ∅` **then** break
- encode `elems` using `xf_{best}`
- remove `xf_{best}` from `xforms`
reducing preprocessing cost
(and a more in-depth look at substructure detection)

- **transformations considered:**
  - HORIZ
  - LINEAR (4)
  - ALL (18)

```python
while True do
    xf_best ← select_best(xforms, elems)
    if xf_best == ∅ then break
    encode elems using xf_best
    remove xf_best from xforms
```

in: `elems` (matrix elements)
in: `xforms` (set of transformations)
reducing preprocessing cost
(and a more in-depth look at substructure detection)

- **transformations considered:**
  - HORIZ  
  - LINEAR (4)  
  - ALL (18)

- **preprocessing windows:**
  - sorting is $O(n \log n)$

```
select_best(xforms, elems):
    xf_best ← ∅;
    score_max ← 0;
    foreach xf in xforms do
        substr ← detect(xf, elems);
        score ← get_score(substr);
        if score > score_max then
            xf_best = xf;
            score_max = score;
    return xf_best

detect(xf, elems):
    elems ← xf(elems)
    Sort(elems)
    substr ← horiz_detector(elems)
    elems ← xf^{-1}(elems)
    return substr
```
reducing preprocessing cost
(and a more in-depth look at substructure detection)

- **transformations considered:**
  - HORIZ  
  - LINEAR (4)  
  - ALL (18)

- **preprocessing windows:**
  - sorting is $O(n \log n)$
  - we keep complexity to $O(nnz)$ by running detection in windows of constant size $w$

```plaintext
select_best(xforms, elems):
    xf_best ← ∅;
    score_max ← 0;
    foreach xf in xforms do
        substr ← detect(xf, elems);
        score ← get_score(substr);
        if score > score_max then
            xf_best = xf;
            score_max = score;
    return xf_best

detect(xf, elems):
    substr ← ∅
    for i ← 1 to $\left\lceil \frac{nnz}{w} \right\rceil$ do
        welems ← window(elems, w)
        welems ← f(welems)
        Sort(welems)
        substr += horiz_detector(elems)
        welems ← $f^{-1}(welems)$
    return substr
```


reducing preprocessing cost
(and a more in-depth look at substructure detection)

- transformations considered:
  - HORIZ  · LINEAR (4)  · ALL (18)

- preprocesing windows:
  - sorting is $\mathcal{O}(n \log n)$
  - we keep complexity to $\mathcal{O}(nnz)$ by running detection in windows of constant size $w$

- sampling:

```
select_best(xforms, elems):
    xf_best ← ∅;
    score_max ← 0;
    foreach xf in xforms do
        substr ← detect(xf, elems);
        score ← get_score(substr);
        if score > score_max then
            xf_best = xf;
            score_max = score;
    return xf_best

detect(xf, elems):
    substr ← ∅
    for i ← 1 to $\left\lceil \frac{nnz}{w} \right\rceil$ do
        welems ← window(elems, w);
        welems ← f(welems);
        Sort(welems);
        substr += horiz_detector(elems);
        welems ← $f^{-1}(welems)$;
    return substr
```
reducing preprocessing cost
(and a more in-depth look at substructure detection)

- transformations considered:
  · HORIZ  · LINEAR (4)  · ALL (18)

- preprocessing windows:
  - sorting is $O(n \log n)$
  - we keep complexity to $O(nnz)$ by running detection in windows of constant size $w$

- sampling:
  detection on a constant number of windows (uniformly distributed)

select_best(xforms, elems):
- \( xf_{\text{best}} \leftarrow \emptyset \);
- \( score_{\text{max}} \leftarrow 0 \);
- \( \textbf{foreach} \ xf \text{ in } xforms \text{ \bf do} \)
  - \( substr \leftarrow \text{detect}(xf, \text{elems}) \);
  - \( score \leftarrow \text{get_score}(substr) \);
  - \( \textbf{if} \ score > score_{\text{max}} \text{ then} \)
    - \( xf_{\text{best}} = xf \);
    - \( score_{\text{max}} = score \);
- \( \text{return } xf_{\text{best}} \)

detect(xf, elems):
- \( substr \leftarrow \emptyset \)
- \( \textbf{foreach } i \text{ in } \text{samples} \text{ \bf do} \)
  - \( \text{welems} \leftarrow \text{window}(\text{elems}, w) \)
  - \( \text{welems} \leftarrow f(\text{welems}) \)
  - \( \text{Sort(welems)} \)
  - \( substr += \text{horiz_detector}(\text{elems}) \)
  - \( \text{welems} \leftarrow f^{-1}(\text{welems}) \)
- \( \text{return } substr \)
Experimental evaluation

- **Machines:**
  - Harpertown
  - Dunnington

- 15 matrices from real-world applications
- compare against:
  - CSR
  - BCSR (select the best performing block)
- double (64-bit) floating point values
Experimental results: performance improvement
(over multithreaded CSR)

For 8 cores:
- average speedup: 2.21 (33% better than CSR)
- BCSR outperforms CSX only for one matrix
- no matrix with slowdown for CSX
Experimental results: sampling

CSX average performance improvement vs preprocessing cost
Conclusions & future work

CSX:
- aggressive index data compression to optimize SpMxV
- supports arbitrary regularities
- tunable preprocessing cost
- code available at: http://www.cslab.ece.ntua.gr/csx/
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can SpMxV scale?

CSR:

| index data (32-bit) | value data (64-bit) |

- index data compression → diminishing returns
  (since value data dominate)
Conclusions & future work

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Currently working on:
- improving CSX (e.g., NUMA support, improved heuristics)
- integrating CSX on ELMER (Open Source Finite Element Software)
- power efficiency considerations
Thank you!
Questions?

The First Rule of Program Optimization:
Don’t do it.

The Second Rule of Program Optimization (for experts only!):
Don’t do it yet.

- Michael A. Jackson
Backup slides
Application classes
(based on their performance on shared memory systems)
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(based on their performance on shared memory systems)

✅ Good scalability

✅ temporal locality

✅ no dependencies

main memory (or off-chip cache)
Application classes
(based on their performance on shared memory systems)

✗ Applications with intensive memory accesses
  ✓ (very) poor temporal locality
  ✓ high memory-to-computation ratio
  ✓ limited scalability due to contention on memory

main memory (or off-chip cache)
Improving performance using compression
exchange memory cycles for CPU cycles

serial

parallel (4 cores)


c
m

c

c

Decompression cost amortization
Improving performance using compression exchange memory cycles for CPU cycles
Improving performance using compression
exchange memory cycles for CPU cycles

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decompression cost amortization
optimizing SpMxV using index compression
(connection with previous work)

- index data: column indices
optimizing SpMxV using index compression
(connection with previous work)

- index data: column indices

- delta encoding ([Willcock and Lumsdaine ’06]):
  instead of $c_i$, store $\delta_i = c_i - c_{i-1}$
  $\Rightarrow \delta_i \leq c_i \Rightarrow$ (potentially) less space per index
optimizing SpMxV using index compression
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- CSR-DU ([Kourtis et al. ’08]): coarse-grained delta encoding
optimizing SpMxV using index compression
(connection with previous work)

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- CSR-DU ([Kourtis et al. ’08]): coarse-grained delta encoding

- **CSX**: (more) aggressive compression by supporting units with arbitrary regularities ($O(1)$ space)